## Alexey L. Gorodentsev

## One semester course «Sheaves And Homological Algebra»

This is an introduction to the theory of sheaves and supplying homological algebra — commonly used technique for handling locally defined objects<sup>1</sup> on a manifold *X*. In algebraic and/or differential geometry and topology it allows to produce global geometric and topological invariants of *X* from those local data. In non-commutative geometry it gives various geometric style invariants of categories equipped with Grothendieck topologies.

**Duration**: September – December, 2015.

## Recommended textbooks:

[D] V. I. Danilov. Cohomology of Algebraic Manifolds. English version in: Algebraic Geometry II: Cohomology of Algebraic Varieties. Algebraic Surfaces. Encyclopaedia of Mathematical Sciences. Book 35. Springer (1995). Russian version in: Алгебраическая геометрия – 2. Итоги науки и техники. Совр. пробл. математ. Фунд. напр. Т. 35. Москва, ВИНИТИ, 1989, с. 5 – 130.

[GH] P. Griffiths, J. Harris. Principles of Algebraic Geometry.

[GM] S. I. Gelfand, Yu. I. Manin. Methods of Homological Algebra. Part I.

[I] B. Iversen. Cohomology of Sheaves.

[W] C. A. Weibel. An Introduction to Homological Algebra.

**Prerequisites**: the first 3 semesters (6 modules) of the standard courses «Algebra», «Calculus», and «Geometry/Topology» given at our faculty or at IUM.

## Preliminary program:

- 1. Categories, functors, pre-sheaves. The main working examples: open sets of a topology and simplicial sets. Cateory of functors, Yoneda's lemma. Adjoint functors. (Co) limits of diagrams. ([GM], [W])
- 2. Sheaves on topological spaces. Stalks and the étalé space of a sheaf. Sheafification. Pull back and push forward. Abelian sheaves. ([I], [GM])
- 3. Complexes and (co)homologies. Long exact sequence of cohomologies. The Koszul complexes. Cohomologies and filtered colimits. Spectral sequences of filtered complexes, bicomplexes, and exact couples. ([GM], [D], [W])
- 4. Global sections, flabby sheaves, and the Godement resolution. Sheaf cohomology and hypercohomology. Acyclic resolutions. The Mayer Vietoris exact sequence and Čech's resolution. Acyclic coverings and Cartan's criterion for acyclicity. The Čech cohomologies. ([I], [D])
- 5. Fine and soft sheaves. The sheaves of differential forms, the Poincaré lemma and the DeRham theorem. ([GM], [GH], [D])
- 6. Higher direct images. The Leray spectral sequence. ([I], [D], [GH])
- 7. (If the time allows.) Coherent sheaves in algebraic geometry: examples and applications. Acyclicity of affine varieties. Cohomologies of invertible sheaves on projective spaces. ([D])
- 8. (If the time allows.) Grothendieck topologies and sheaves on sites. ([GM])

<sup>&</sup>lt;sup>1</sup>e.g. functions with restricted domains of definition, local sections of vector bundles, locally defined continuous mappings etc